

Numerical research on mechanisms of earth fissure formation in Su-Xi-Chang area, China, based on extended finite element method

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Abstract

Introduction

With ground-water extraction activity becoming a vital water supply during urbanization, lots of attentions are paid on earth fissure under pumping situation since it gradually turns into a significant urban geologic hazard, especially near some metropolis. Escalating pumping activities imperceptibly undermine the previous ground-water balance in the nearby district, causing land subsidence, and sometimes earth fissures. When such intensive pumping activities take place in a populated area, billions were cost as earth fissures bring damage to infrastructures and jeopardize ground-water quality.

First earth fissure associated with pumping were reported in south-central Arizona [1]. Apart from tectonic influence, earth fissure activities in many areas showed strong temporal coincidence with pumpage fluctuation [2] and spatial coincidence with edge of the subsidence zone [3]. Naturally, they are presumed to be results of pumping activities and associated land subsidence. But still, due to difficulties in observing and monitoring deep soil fissuring process and the usual complexity of geological conditions in large-scale alluvial stratum, most earth fissures researches are largely limited to the qualitative ground surface field investigation or the correlation between earth fissure and other factor such as subsidence, while the precise mechanics of the formation and propagation of pumping-induced earth fissure is still remain hypothetical in details.

On the other hand, although numerical simulation has become an effective tool to analyze mechanical behavior of soil under variable situations, earth fissure simulation research is still significantly insufficient comparing to other soil mechanics fields because of the difficulties in simulating crack behaviors. As crack formation and propagation progress involves strong discontinuity and highly non-linearity, continuum mechanics shows inadequacy. With the development of failure mechanics as well as its implementation in numerical simulation, crack behaviors become feasible to simulate without enormous calculation. Generally, in an earth fissure numerical analysis it might be expected that an ideal simulation approach should contain following characters:

1. None or little mesh dependency and relatively low computer time cost. Earth fissure problem usually involves large-span spatial structure analysis, which causes a relatively coarse mesh and lots of computational time. So any numerical approach used might face a severe challenge on mesh independency and extra computer time for crack analysis. Otherwise it may lead to a mesh biased results or an burdensome elapsed time.

2. Ability to arbitrarily crack propagate. Propagation is vital to analyze further behavior of crack. In most scenarios, the path of earth fissure propagation is unknown. So during simulation the crack should be allowed to propagate at arbitrary direction and magnitude.

3. Using at least one feasible fracture criterion. Here feasible means such fracture criterion value is easily calculated in simulation and easily measured in laboratory test. Regard to fundamental assump-

tions of fracture mechanics, the stress and strain at crack tip is infinity and dramatically changing in the neighborhood, so a better solution is using energy-based criterion such as J- integral or fracture energy.

So far several approaches are able to achieve this goal. Standard finite element method can describe crack by adopting Smear Crack Model (SCM) or Fictitious Crack Model (FCM). But more effective are new methods designed especially for solving discontinuity problems, namely Element Free Galerkin Method (EFGM), Discrete Element Method (DEM), Displacement Discontinuity Method (DDM), and Extended Finite Element Method (XFEM). Naturally, each method has different scopes and properties. SCM and FCM are easiest to build but have several critical limitations. SCM has poor support to Mode II crack and more or less mesh dependency, even if an energy-based criterion is used. FCM requires either the cracks propagating along the edge of elements or constantly remeshing during propagation. It might be more effective on a stationary crack analysis rather than propagation problems. EFGM along with other element-free methods (EFM) lacks a solid theory foundation and holds some uncertainty. The influential radius and weight function in EFM is critical, but their physical significance and determination are still in dispute. DEM and DDM treat structure as a bunch of interactional blocks and discontinuity can happen only between block elements, which usually apply to discrete block medium such as rocks and sand grains at meso-level. In contract, XFEM is particularly designed to describe macroscopic discontinuity in continuum medium with high accuracy and less meshing, originally presented by Belytschko [9]. XFEM inherits all advantages of standard finite element method and is especially well compatible when a standard finite element codes exists. XFEM has already been applied to soil mechanics in several fields, mostly on hydraulic fracture researches. Also it can be an ideal approach to simulate any discontinuity in geotechnical materials such as earth fissure. One of few researches concerning earth fissure propagation based on XFEM has been presented by Hernandez [10].

Numerical simulations are quite rare in earth fissure analyses, and most of them are based on

SCM/FCM or simply continuum mechanics, mainly focused on locating latent tension zone in aquifer system.

Geology background in Su-Xi-Chang area

Su-Xi-Change area refers to the urban circle of Suzhou City, Wuxi City and Changzhou City in Jiangsu Province, China, with Jiangyin City, Zhangjiagang City, Changshu City, Kunshan City, Taicang City and Wujiang City also under jurisdiction. SXC area locates at Yangtze Plate in the core of Tai Lake basin which is one of the most developed regions in China, covering an area of about 12000km². Surveys presented in the 90s by Geological Survey of Jiangsu Province showed no evidence of any major tectonization during the Quaternary and Holocene period in SXC area, so the strata in Quaternary are relatively horizontal. However, the underlying bedrock formed at an earlier time is dramatically fluctuant comparing to a fairly plain surface. Figure 1 depicts a geological diagrammatic section of aquifer system from northwest area near Changzhou City to southeast area near Suzhou City. Top and floor in one aquifer are depicted in same color, and subsidence is depicted as purple layer. The depth is amplified 70 times and the subsidence is amplified 3500 times for better recognition. As showed, underlying bedrock uplifts to outcrop on the surface in the west-center part near Wuxi City and Changzhou City, forming two series of NE/NNE trending ridges and intermittently cutting off hydraulic connection of several aquifers. Then it gradually sinks down to the depth to both sides south-eastward and north-westwards. Upon bedrock massive fluvial silt from Yangtze River, lacustrine deposit from Tai Lake and marine deposit during transgression has been accepted in this area, constituting an aquifer system with great thickness and complex properties, which can also be broadly divided as one phreatic aquifer, three confined aquifers and aquitards in between. Generally, the water table of phreatic aquifer is shallowly buried, at only 3-5m below surface. The 1st confined aquifer is

buried at about 80m below, mainly distributes in the east part, reaching a maximum thickness of 70m and getting thinner westwards, eventually missing in the middle part. Additionally, the northwest area near Changzhou City is also covered by 1st confined aquifer with rather low thickness. The 2nd confined aquifer is buried at about 120m below and covers most area except for outcropped zones, with thickness ranging from 10 to 60m. It becomes the main resource of ground-water supply in SXC area and suffers most severe pumping. The 3rd confined aquifer is buried at about 170m below with low thickness, mostly cut off by uplifting bedrock, and distributes only on limited parts in the east and northwest.

With rapid growth of population and economy, aquifer system in SXC area has experienced constant and severe extraction since the 70s during massive urbanization to satisfy expanding water demands. Pumping activities take place mainly in the 2nd confined aquifer because of its rich storage and wide distribution, leading to a drop down of piezometric head 70m at most. Although since 2000, a strict prohibition on pumping activity in all SXC area had been executed and ground-water recharging has been carried out in some areas, associated phenomenon including land subsidence and earth fissures had already widely sprouted out and become major urban geological hazards in this area. Figure 2 depicts the distribution of cumulative subsidence (to 2002) and earth fissure in SXC area. Subsidence in SXC area originally centralized in downtowns of the three major cities of Suzhou, Wuxi and Changzhou, and soon joined together with the expansion of subsidence funnels. With great efforts in prohibition on pumping activity and recharging, the rate of subsidence has been contained but the cumulative subsidence didn't rebound due to plasticity of soil deformation. Up to 2005, the maximum cumulative subsidence lies in Xishan district, Wuxi, reaching over 2000mm. Meanwhile, earth fissure in this area was firstly observed in late 80s in Wujin, Changzhou City and then widely emerged in 90s in dozens of locations in the west-center part. Even after years of prohibition of ground-water pumping, some earth fissures still remain active due to different reasons such as hysteresis of soil deformation, bringing a great number of economic damage.

Comparing strata and earth fissure distribution, the fissure zones usually show several features as follow: (1) Lack of 1st and 3rd confined aquifers. The 2nd confined aquifer becomes the only source of ground-water. There are few exceptions near Zhangjiagang City, where both 1st and 2nd confined aquifers have suffered severe pumping. (2) Shallow bedrock buried. Fissures all concentrated in the west-center part of SXC area with bedrock buried at about 80-120m below surface. (3) Paralleling with subsidence contours or nearby outcropped bedrock. As the uplifting bedrocks have formed two NE/NNE trending ridges, one from Changzhou City to Jiangyin City and one from Wuxi City to Zhangjiagang City, the most observed earth fissure in SXC area are paralleling with them or the edge of subsidence funnel.

Obviously, earth fissures in this area show quite strong relevance to ground-water decline, subsidence funnels and bedrock outcrops temporally or spatially, and some fissures show considerable opening and vertical offset. Several investigations have been carried out focusing on this phenomenon (). Based on these facts, differential subsidence mechanism is usually favorable in these investigations on earth fissure formation in SXC area.

Conceptual models in SXC area

As mentioned earlier, interference of tectonization is minimized since Quaternary in SXC area. It is not likely to form a large fault in Quaternary sediment. On the other hand, the underlying bedrock is highly fluctuant, so bedrock ridges and abrupt change in alluvial thickness is often the case in this area. So it is crucial to understand relations between earth fissure and pumping activities, subsidence or geological morphology. Although it is commonly accepted that earth fissures usually possess strong temporally or spatially relative to these geological factors, especially in SXC area, but how they exactly contribute to earth fissure formation is usually confounded. For example, bedrock morphology is often coincident with shape of subsidence funnel so it is difficult to distinguish influence of these two factors. Based on different field case, several theories are made in recent decades.

1. Originated by differential subsidence. Jachen and Holzer [4] presented bending beam theory based on the fact that the locations of earth fissures usually coincident with points of maximum convex-upward curvature of buried bedrock surface in earlier survey, so the overlying surface stratum act like a beam bent down by regional subsidence. Holzer [6] also found that in some cases in Arizona, California, and Nevada, fissures are associated with zones of localized differential subsidence. This is slightly different from regional differential subsidence theory because localized differential subsidence theory needs an abrupt change in vertical displacement, while in regional differential subsidence theory localized differential subsidence in fissure zone may be very small and the fissure occurs at the maximum point of a widespread tensile zone. Such theory attributes earth fissure to considerable value of regional differential subsidence and can only applied to fissures forming at surface and propagating downwards.

2. Originated by hydraulic force. In some cases which fissures generate in depth and propagate upwards to the surface or even occur in areas without significant land subsidence, a hydraulic theory was proposed. This theory takes earth fissure as a product of seepage force, first discussed analytically by Lofgren [7]. Lofgren suggests that the in situ driving force at any investigated point in aquifer should relate to cumulative seepage force between the discharging well and the investigated point, rather than just the seepage force at that point. So a pumping well tends to yield a significant tensile stress at shallow stratum far away from the well with little localized subsidence and provide a favorite environment for fissure formation. This established a solid foundation for hydraulic theory. Later Helm modified hydraulic theory by distinguishing seepage force and actual driving force on the solid particles [8].

3. Originated by water-table changing in phreatic aquifer. Budhu presented a new theory on fissure formation under water-table changing in phreatic aquifer situation. Budhu proposed that fissures are more likely triggered by combination of bending and shearing rather than just bending [5]. This theory discarded equality between the shear stresses on vertical and horizontal planes, which is a fundamental

assumption in solid mechanics. With this theory, an earth fissure can form at the surface and propagate downwards or it can form beneath the surface and propagate upwards. And also, it can be an explanation to the fact that fissures tend to become more active in flood season or rainy season, which is common in many earth fissure zones, such as SXC area.

It should be noted that none of these theories are all-inclusive. Some other pumping-related causes such as shrinkage or landslide are beyond scope of this article because they usually refer to a very particular situation. For better understanding on relation between earth fissure, subsidence, and geological morphology in SXC area, we examined geological cross sections of all earth fissures in SXC area. Based on theories above, four strata scenarios are summarized, illustrated in Fig 3.

First is an upwards bedrock ridge buried below aquifers (Fig 3-a). The top of buried ridge is dozens of meters below surface. Pumping mainly takes place in a deep confined aquifer. This scenario usually happens in heavy subsidence zone with shallow buried bedrock near Wuxi City and Changzhou City. The second scenario is that fissure occurs in the very edge of soil near bedrock outcrops (Fig3-b). Localized subsidence is usually very low as the underlying bedrock slope prevented further vertical displacement. This scenario mainly distributes aside the outcropped part of the NE/NEE trending ridge between Wuxi City and Zhangjiagang City. The first two scenarios are predominant within earth fissure cases in SXC area and often seen as a strong evidence of differential subsidence mechanism in most investigations because of the coincidence between fissures and uplifting bedrock. In the third scenario a bedrock terrace is observed (Fig3-c). The fissure locates above the slope shoulder in the upper platform. This scenario happens in the south of Changzhou City at a relatively marginal position of subsidence funnel. The last scenario is barely related to uplifting bedrock (Fig3-d). Contrarily, in the case depicted in Fig3-d, fissure lies on the northwest side of subsidence funnel with an uplifting bedrock in the opposite side. Meanwhile, change in aquitards thickness can be observed under fissure zone. This scenario is near Zhangjiagang City.

Based on geological section maps of earth fissure, two-dimensional numerical analyses are performed. All cases are simplified into plane strain models, illustrated in Fig 4. Boundaries are defined as follow. The other end of stratum is set as a slope, quite a distance away from pumping well to avoid unnecessary influence, except for the second model. The far-end of pumped aquifer is boundary of no excess pore-water pressure, supplying a horizontal flow through the aquifer to the well. Pumping wells are not distributed all over subsidence zone like real scenario; instead, they are only placed in the central of subsidence funnel. The pumpage is accommodated correspondingly to yield a reasonable surface subsidence.

The four models share the same stratum sequences with one phreatic aquifer and one confined aquifers as well as aquitards in between, but the bedrock in the fourth model buried deeper than other ones, depicted in Fig 4-e. in the first model (Fig 4-a), uplifting bedrock cut off the lower part of the 2nd confined aquifer. Since earth fissures lie in the middle of subsidence zone, pumping wells are placed on both side of fissure. Strata in the second model suffer no disturbance and remain horizontal. The 2nd confined aquifer is pumped. In the third model, pumping occurs in the 1st confined aquifer with 2nd confined aquifer absence near pumping well. The fourth model contains one phreatic aquifer and one confined aquifer (Fig 4-f). The aquitards beneath the pumped aquifer is cut by a terrace (Fig 4-d). Piezometric heads in all aquifers are equal to hydrostatic pressure, with water-table at 3m below surface.

Numerical approach base on XFEM

A mesh is created containing mostly quadrangles and few triangles. Displacement and pore pressure are calculated simultaneously based on Biot 2-D consolidation theory:

Where G is shear modulus, w_x and w_y are displacement in two directions, ν is Poisson ratio, V is volumetric strain, and w are unit weight of bulk and water, K is the permeability coefficient, Δ is lapla-

cian operator. Duncan-Chang model is employed as constitutive model in soil under pressure and elastic model in soil under tension. The crack is considered as a fold-line-like segment chain independent of mesh. When the fracture criterion exceeds critical value, a new segment with preset length is added to the previous chain.

In order to model strong discontinuity in numerical simulation, the shape of earth fissure should be described. Normally it can be achieved inter-elementally like FCM. In doing this, the shape of discontinuity explicitly depends on elements shape and position. If an arbitrary propagation path is purposed regardless of previous elements edges, continuous remeshing to adapt crack shape changing is required. This leads to heavy computation efforts on remesh and projection between variables before and after such remesh []. Another strategy is to describe crack intra-elementally. This is the core idea to XFEM and avoids all difficulties and limitations mentioned in inter-elementally methods.

XFEM describes crack shape implicitly by introducing extra shape function enrichment based on partition of unity. This enrichment can be regarded as an accurate approximation on discontinuity part of displacement while the normal shape function used in standard FEM can be regarded as an approximation on the continuity part. This strategy is similar to p-version FEM but the latter only applies when variables can be well fitted by known smooth functions with high order. The general displacement pattern used in XFEM is:

Where i is the set of all nodes in research area, j is the set of all nodes which belong to elements cut across by crack (shown as square nodes in Fig. 5) and k is the set of nodes belonging to the element pieced by crack tip (circular nodes in Fig. 5). U_i are the classical degree in FEM. A_j and b_j are the additional degree of freedom. $N_i(x)$, $N_j(x)$ and $N_k(x)$ are called partition of unity functions which are not necessarily same but usually determined as the normal shape function for simplicity. The first term in the right hand constitutes classical continuous displacement pattern in standard FEM. The second term refers to a step function enrichment to describe an abrupt change of displacement across the crack. Earlier form of $H(x)$

is a standard Heaviside function. However in order to maintain partition of unity condition in adjacent elements, $H(x)$ here is defined as:

The third term in right hand of equation (1) refers to linear elastic asymptotic function enrichments to crack tips, intending to approximate the dramatically changing displacement pattern near a crack tip. The specific forms of function $f_n(x)$ are chosen based on linear elastic fracture mechanics. If a plane problem is involved, n is usually taken as 4. And $f_n(x)$ are given by:

Where r and θ are the radius and angle in local polar co-ordinates at crack tip. Such enrichments not only allow FEM using analytical solution to reach a better approximation of displacement pattern near tips, but also considerably reduce necessity for fine mesh near the crack tips.

Normal numerical integration strategy is no longer competent in elements along crack, since the original Gaussian points may be insufficient. Putting aside the integral precision, if a crack crosses through an element like Fig 6(a), enriched degrees of freedom in node j and k lose support from Gaussian points (represented by small crossings) because according to equal (2), only Gaussian points on the other side of crack yield non-zero $H(x)$. So the bottom line of numerical integration strategy is that Gaussian points should at least distribute on both side of crack.

A simple decomposed numerical integration strategy depicted in Fig 6(b-c) is employed. In elements cut by crack or pieced by tip, integration domain is subdivided as several triangles. And the topology relationships between crack, nodes and Gauss points are calculated by level set functions [1]. The mode I and mode II intensity factor K_I and K_{II} are determined by domain form of the interaction integral [2]. The possible propagation direction can be determined by maximum hoop stress criterion as:

θ_0 is angle of propagation direction with respect to current direction of crack tip. And meanwhile a corresponding equivalent stress intensity factor is determined as fracture criterion:

When K_{eq} exceeds the fracture toughness K_{IC} determined by experiments, a piece of segment with pre-defined length at angle θ_0 is added to current segment chain. The crack propagated. A test illustrated in

Fig 7 was carried out to check validity of XFEM. J
LINEAR ELASTIC CONVERGENCE STUDY

Simulation results