

Mechanism research of pumping-induced earth fissure formation in Su-Xi-Chang area based on XFEM

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Abstract

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1. Introduction

SXC area is one of areas most severely affected by earth fissures in China. During past decade, millions are cost to investigate and alleviate the regional earth fissure activities. The detailed situation of earth fissure hazard in SXC area can be found in Zhang et al. (2008). Even though earth fissure has become a world-wide urban hazard only since firstly reported in USA in recent decades, it significantly poses a thread to urban constructions and jeopardizes groundwater quality. In order to understand and control earth fissure hazard, theories have been made to explain the mechanism of earth fissure formation. However, because of the difficulty in monitoring the underground situation, the mechanism of earth fissure formation still remains hypothetical in details.

On the other hand, numerical simulation has been proved promising in geotechnical researches. The obstruction of the earth fissure simulation development lies in

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the difficulty of strong discontinuities simulation. Conventional numerical method used in geotechnical researches such as finite element method or finite difference method show inadequacy to solve such problems. In simulation of earth fissure formation, a propagation of earth fissure with arbitrary and complex path is expected. Conventional numerical method such as finite element method face the difficulty in describing strong discontinuity. Earlier attempts to solve such problems based on conventional finite element method involved either a constraint that cracks could only propagate along the edge of elements or a constant remesh around the new cracked part during propagation (Bouchard et al., 2003). Methods based on the concept of material damage are an alternative but contain significant limitations when mode II propagation is confronted, such as Crack Band Method(CBM)(Bažant and Oh, 1983). The Extended Finite Element Method ("XFEM" for short) is a method specially designed for discontinuity problems within the finite element method framework, proposed by Belytschko and Black (1999). It seems suitable to describe earth fissure formation and propagation by XFEM.

2. Extended finite element method

On the basis of the partition of unity finite element method (PUFEM) (Melenk and Babuška, 1996), Belytschko and Black (1999) proposed the extended finite element method(XFEM). Since then, XFEM has been proved to be an reliable tool to simulation cracks' propagation.

According to PUFEM, the approximation of function u on an open set Ω can be written as:

$$u = \sum_i \varphi_i v_i \tag{1}$$

where v_i is the local approximation function on Ω_i , which are the open cover of Ω satisfying:

$$card\{i|\mathbf{x} \in \Omega_i\} \leq n \quad (2)$$

φ_i is a partition of unity on Ω_i satisfying:

$$\sum_{i=1}^n \varphi_i = 1 \quad (3)$$

$$\|\varphi_i\|_{L^\infty \Omega} \leq C_\infty \quad (4)$$

where n and C_∞ are constants. The detailed derivation process and convergence proof can be found in Moran and Shih (1987).

If we consider the local approximation function v_i as:

$$v_i = span(p_i^j)(j = 1, 2, \dots, m) \quad (5)$$

then Eqn. (1) can be written as:

$$u = \sum_{i=1}^n \sum_{j=1}^m a_i^j \varphi_i p_i^j, a_i^j \in \mathbb{R} \quad (6)$$

a_i^j here can be considered as the generalized degrees of freedom. p_i^j can be customized by a priori knowledge of the solution for a better approximation.

Eqn. (6) can be interpreted as a unified form for most numerical methods. Take p_i^j as 1 and φ_i as normal shape functions, Eqn. (6) reduces to the approximation function of conventional finite element method. In XFEM, in order to describe

strong discontinuity and near-tip asymptotic field, p_i^j takes the form of an extrinsic enrichment and the approximation can be written as (Belytschko and Black, 1999):

$$u = \sum_{i=1}^n N_i(\mathbf{X}) \left(u_i + H(\mathbf{X}) a_i + \sum_{j=1}^m \gamma^j(\mathbf{X}) b_i^j \right) \quad (7)$$

here $H(x)$ refers to a jump function enrichment to describe an abrupt change of displacement across the crack:

$$H(\mathbf{X}) = \text{sign}(\phi(\mathbf{X})) \quad (8)$$

where

$$\text{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases} \quad (9)$$

and $\phi(\mathbf{X})$ is the signed distance function to the crack called "level set function", which is negative on one side of crack face and positive on the other side.

The specific forms of function $\gamma^j(\mathbf{X})$ are the terms chosen from the Westergaard solution for the near-tip asymptotic field (Belytschko et al., 1994). If a plane problem is involved, m is taken as 4, and $\gamma^j(\mathbf{X})$ are usually calculated in polar coordinate form:

$$\gamma^{(1-4)}(r, \theta) \equiv \left\{ \sqrt{r} \sin\left(\frac{\theta}{2}\right), \sqrt{r} \cos\left(\frac{\theta}{2}\right), \sqrt{r} \sin\left(\frac{\theta}{2}\right) \sin \theta, \sqrt{r} \cos\left(\frac{\theta}{2}\right) \cos \theta \right\} \quad (10)$$

where r and θ are the radius and angle in the local polar coordinate at the crack tip, respectively.

In practice, in order to minimize influence of such extrinsic enrichment on un-

enriched node, a more suitable form of approximation function is used as:

$$u(\mathbf{X}) = \sum_{i \in N^{std}} N_i(\mathbf{X}) u_i + \sum_{j \in N^{sign}} N_j(\mathbf{X}) [H(\mathbf{X}) - H(\mathbf{X}_j)] a_j + \sum_{k \in N^{tip}} N_k(\mathbf{X}) \left[\sum_{m=1}^n [\gamma_k^m(\mathbf{X}) - \gamma_k^m(\mathbf{X}_k)] b_k^m \right] \quad (11)$$

where $\mathbf{X}_j$ and $\mathbf{X}_k$ are the node coordinates; N^{std} is the set of all nodes in the research domain; N^{sign} is the set of all nodes which belong to the elements cut across by a crack (square nodes in Fig. (1)); N^{tip} is the set of nodes belonging to the element pieced by a crack tip (circular nodes in Fig. (1)); u_i are the classical degrees of freedom in FEM; a_j and b_k^m are the corresponding additional degrees of freedom associated with the enrichments; N_i , N_j and N_k are called partition of unity functions. They are not necessarily the same but usually all set as the normal shape functions for simplicity.

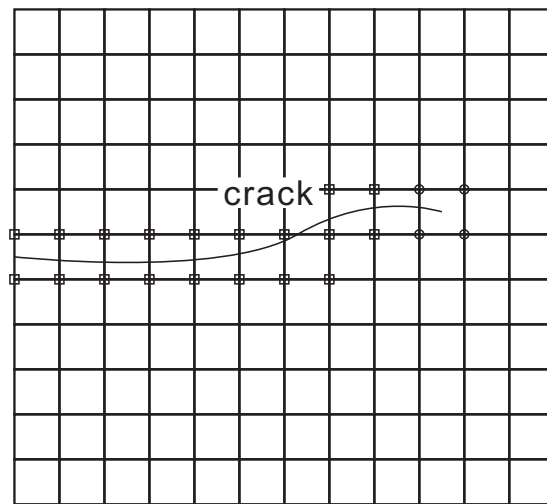


Figure 1: Numerical integration strategy in XFEM

Conventional numerical integration strategy is no longer competent in the elements penetrated by cracks (Mohammadi, 2011). Regardless of the integral precision,

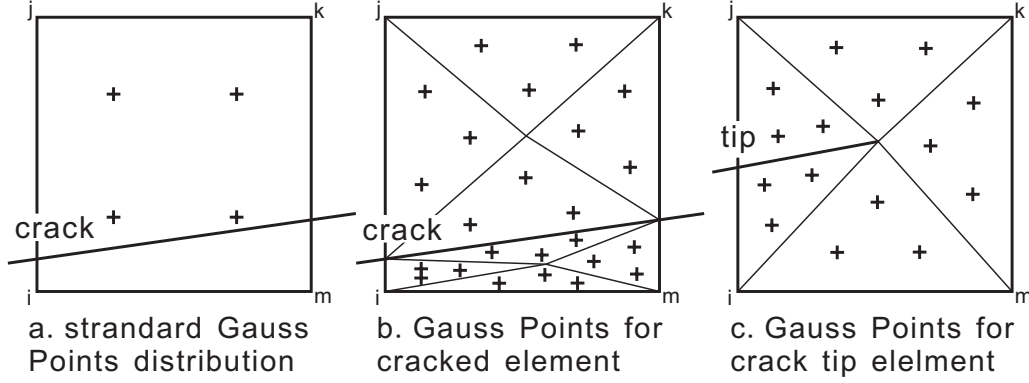


Figure 2: Stratigraphic section of aquifer system trending NE across SXC area

if a crack crosses through an element with all Gaussian points on one side as plotted in Fig. (2)(a), nodes j and k will not be enriched. Consequently, the basic requirement is that the Gaussian points within a cracked element should be at least distributed on both sides of a crack. A simple integration strategy depicted in Fig. (2)b-c is employed (Dolbow et al., 2000). In the elements totally penetrated by a crack or pieced by a crack tip, the integration domain is subdivided as several triangles. The topology relationships between the crack, the nodes and the Gauss points are calculated by level set functions.

Normally, earth fissures are usually categorized into tensile fissures and shearing fissures. So both mode I and mode II propagation should be considered during simulation. Generally, the maximum tangential stress criterion (MTS) is the most widely used criterion in crack simulation for its simplicity (Erdogan and Sih, 1963). The minimum strain-energy-density criterion (S-criterion) (Sih, 1974) and the maximum strain energy criterion (G-criterion) (Nuismer, 1975) are also used in researches. However, MTS and S-criterion fail to predict a reasonable propagation direction when a mode II crack is confronted (Shen and Stephansson, 1994). G-criterion and its variation seems to be suitable to act as a unified criterion for both mode I and

mode II propagation, but they are quite difficult to calculate numerically.

A combined fracture criterion containing two sub-criteria is employed in this research. Stress intensity factors K_I and K_{II} are obtained by the interaction integral (Moran and Shih, 1987). The critical state for propagation is stated by Eqn. (12):

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2} \quad (12)$$

where K_{eq} is the equivalent stress intensity factor. The crack tip opening displacement (CTOD) at the Gauss points on the crack surfaces near the crack tip are used to determine which kind the propagation is. MTS will be applied if the CTOD near crack tip exhibits tension. The propagating direction is determined by Eqn. (13):

$$\theta = 2 \arctan\left(\frac{K_I/K_{II} \pm \sqrt{(K_I/K_{II})^2 + 8}}{4}\right) \quad (13)$$

where θ is the angle of propagation direction with respect to the current direction of the crack tip. In some cases with highly compressive environment such as the direct shear test or a slide, a fissure with zero CTOD near the crack tip but a significant shearing dislocation cannot be driven by the circumferential tension. Then the maximum radial shear stress criterion (MRS) proposed by Lin Lin (1985) is employed instead. This criterion is specifically designed for mode II propagation, assuming that a pure shearing or compressive-shearing fissure will propagate at the direction with the greatest radial shear stress. The radial shear stress state in the vicinity of the crack tip can be expressed as

$$\tau = \frac{1}{2\sqrt{2/r}} \cos \frac{\theta}{2} [K_I \sin \theta + K_{II}(3 \cos \theta - 1)] \quad (14)$$

The direction of the greatest radial shear stress is yielded by enumeration of Eqn. (14) due to the difficulty to get the explicit form of its derivative.

3. Geological background and earth fissure distribution in Su-Xi-Chang area

Su-Xi-Chang ("SXC area" for short) area refers to the urban circle of Suzhou City, Wuxi City and Changzhou City in Jiangsu Province, China, with Jiangyin City, Zhangjiagang City, Changshu City, Kunshan City, Taicang City and Wujiang City also under jurisdiction. SXC area locates at Yangtze Plate in the core of Tai Lake basin which is one of the most developed regions in China, covering an area of about $12000km^2$. Surveys presented in the 90s by Geological Survey of Jiangsu Province showed no evidence of any major tectonization during the Quaternary and Holocene period in SXC area, so the strata in Quaternary are generally horizontal. However, the underlying bedrock formed at an earlier time is dramatically fluctuant comparing to a fairly plain surface.

Fig. (3) depicts a typical stratigraphic section of aquifer system from northwest area near Changzhou City to southeast area near Suzhou City. Each aquifer is depicted as two layers in same color indicating aquifer's top and floor. The bedrock is depicted as the black layer. Subsidence is also depicted here as a purple layer. The depths of each layers are amplified 70 times and the subsidence is amplified 3500 times for better recognition. As showed, underlying bedrock uplifts to outcrop on the surface in the west-center part near Wuxi City and Changzhou City, forming two series of NE/NNE trending ridges and intermittently cutting off hydraulic connection of several aquifers. Then it gradually sinks down to the depth to both sides south-eastward and north-westwards. Upon bedrock massive fluvial silt from

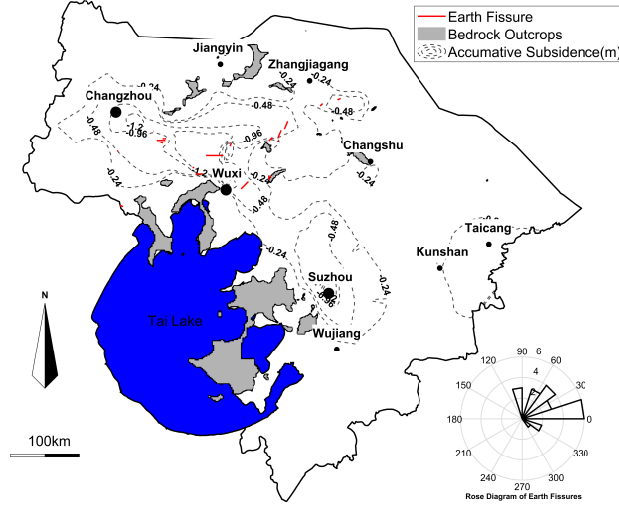


Figure 3: Cumulative subsidence (to 2002) and earth fissure distribution in SXC area

Yangtze River, lacustrine deposit from Tai Lake and marine deposit during transgression has been accepted in this area, constituting an aquifer system with great thickness and complex properties. Such thick sedimentary layer can also be hydrogeologically divided as one phreatic aquifer, three confined aquifers and aquitards in between. Generally, the water table of phreatic aquifer is shallowly buried, at only 3-5m below surface. The 1st confined aquifer is buried at about 80m below, mainly distributes in the east part, reaching a maximum thickness of 70m and getting thinner westwards, eventually missing in the middle part. Additionally, the northwest area near Changzhou City is also covered by 1st confined aquifer with rather low thickness. The 2nd confined aquifer is buried at about 120m below and covers most area except for outcropped zones, with thickness ranging from 10 to 60m. It becomes the main resource of ground-water supply in SXC area and suffers most severe pumping. The 3rd confine aquifer is buried at about 170m below with low thickness,

mostly cut off by uplifting bedrock, and distributes only on limited parts in the east and northwest.

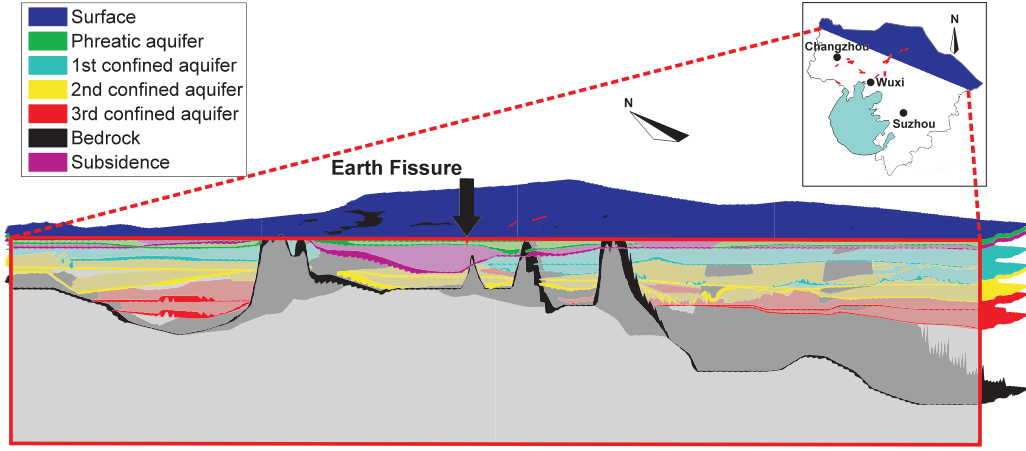


Figure 4: A typical senario of earth fissure stratigraphic section map in Su-Xi-Chang area

With rapid growth of population and economy, aquifer system in SXC area has experienced constant and severe extraction since the 70s during massive urbanization to satisfy expanding water demands. Pumping activities take place mainly in the 2nd confined aquifer for its rich storage and wide distribution, leading to a drop down of piezometric head up to 70m at most. Since 2000, a strict prohibition on pumping activity in all SXC area had been executed and ground-water recharging has been carried out in some areas. However associated phenomenon including land subsidence and earth fissures have already widely sprouted out and become major urban geological hazards in this area. Fig. (3) depicts the distribution of cumulative subsidence (to 2002) and earth fissure in SXC area. Subsidence in SXC area originally centralized in downtowns of the three major cities of Suzhou, Wuxi and Changzhou, and soon joined together with the expansion of subsidence funnels. With efforts in prohibition on pumping activity and recharging, the rate of subsidence has been contained but the cumulative subsidence didn't rebound due to plasticity of soil

deformation. Up to 2005, the maximum cumulative subsidence lies in Xishan district, Wuxi City, reaching over 2000 mm . Meanwhile, earth fissure in this area was firstly observed in late 80s in Wujin and Changzhou City and then widely emerged in 90s in dozens of locations in the west-center part. Even after years of prohibition of groundwater pumping, some earth fissures still remain active due to different reasons such as hysteresis of soil deformation, bringing a great number of economic damage.

Comparing strata and earth fissure distribution, the fissure zones usually show several features as follow: (1) Lack of 1st and 3rd confined aquifers. The 2nd confined aquifer becomes the only source of ground-water. There are a few exceptions near Zhangjiagang City, where both 1st and 2nd confine aquifers have suffered severe pumping. (2) Shallow bedrock buried. Fissures all concentrated in the west-center part of SXC area with bedrock buried at about 80-120m below surface. (3) Paralleling with subsidence contours or nearby outcropped bedrock. As the uplifting bedrocks have formed two NE/NNE trending ridges, one from Changzhou City to Jiangyin City and one from Wuxi City to Zhangjiagang City, the most observed earth fissure in SXC area are paralleling with them or the edge of subsidence funnel.

Obviously, earth fissures in this area show quite strong relevance to ground-water decline, subsidence funnels and bedrock outcrops temporally or spatially, and some fissures show considerable opening and vertical offset. Several investigations have been carried out focusing on this phenomenon (). Based on these facts, differential subsidence mechanism is usually favoured in these investigations on earth fissure formation in SXC area. A more detailed geological background is presented by Zhang et al. (2016).

Conceptual models of earth fissure formation in SXC area

As mentioned earlier, interference of tectonization is minimized since Quaternary in SXC area. It is not likely to form a large fault in Quaternary sediment. On the other hand, the underlying bedrock is highly fluctuant, so bedrock ridges and abrupt change in alluvial thickness is often the case in this area. So it is crucial to understand relations between earth fissure and pumping activities, subsidence or geological morphology. Although it is commonly accepted that earth fissures usually possess strong temporally or spatially relative to these geological factors, especially in SXC area, but how they exactly contribute to earth fissure formation is usually confounded. For example, bedrock morphology is often coincident with shape of subsidence funnel so it is difficult to distinguish influence of these two factors. Base on different field cases all over the world, several theories are made in recent decades.

- 1 Originated by differential subsidence. Jachen and Holzer [4] presented bending beam theory based on the fact that the locations of earth fissures usually coincident with points of maximum convex-upward curvature of buried bedrock surface in earlier survey, so the overlying surface stratum act like a beam bent down by regional subsidence. Holzer [6] also found that in some cases in Arizona, California, and Nevada, fissures are associated with zones of localized differential subsidence. This is slightly different from regional differential subsidence theory because localized differential subsidence theory needs an abrupt change in vertical displacement, while in regional differential subsidence theory localized differential subsidence in fissure zone may be very small and the fissure occurs at the maximum point of a widespread tensile zone. Such theory attributes earth fissure to considerable value of regional differential subsidence and can only applied to fissures forming at surface and propagating downwards.

- 2 Originated by hydraulic force. In some cases which fissures generate in depth and propagate upwards to the surface or even occur in areas without significant land subsidence, a hydraulic theory was proposed. This theory takes earth fissure as a product of seepage force, first discussed analytically by Lofgren [7]. Lofgren suggests that the in situ driving force at any investigated point in aquifer should relate to cumulative seepage force between the discharging well and the investigated point, rather than just the seepage force at that point. So a pumping well tends to yield a significant tensile stress at shallow stratum far away from the well with little localized subsidence and provide a favorable environment for fissure formation. This established a solid foundation for hydraulic theory. Later Helm modified hydraulic theory by distinguishing seepage force and actual driving force on the solid particles [8].
- 3 Originated by water-table changing in phreatic aquifer. Budhu presented a new theory on fissure formation under water-table changing in phreatic aquifer situation. Budhu proposed that fissures are more likely triggered by combination of bending and shearing rather than just bending [5]. This theory discarded equality between the shear stresses on vertical and horizontal planes, which is a fundamental assumption in solid mechanics. With this theory, an earth fissure can form at the surface and propagate downwards or it can form beneath the surface and propagate upwards. And also, it can be an explanation to the fact that fissures tend to become more active in flood season or rainy season, which is common in many earth fissure zones, such as SXC area.

It should be noted that none of these theories are all-inclusive. Some other pumping-related causes such as shrinkage or landslide are beyond scope of this article. They usually link to a very particular situation. For better understanding on

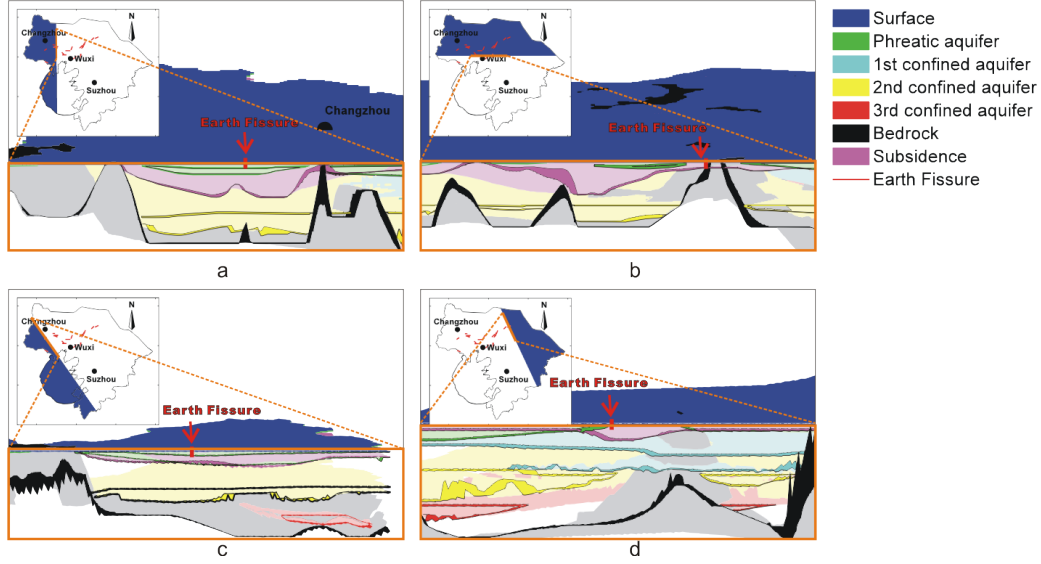


Figure 5: Four types of strata scenario inducing earth fissure formation in SXC area

relation between earth fissure, subsidence, and geological morphology in SXC area, we examined stratigraphic sections of 22 main earth fissures in SXC area. Based on theories above, four strata scenarios are summarized, illustrated in Fig. (5).

First is an upwards bedrock ridge buried below aquifers as Fig. (5)a. The top of buried ridge is dozens of meters below surface. Pumping mainly takes place in a deep confined aquifer. This scenario usually takes place in heavy subsidence zone with an uprising buried bedrock near Wuxi City and Changzhou City. Subsidence (the purple layer) is clearly obstructed by the ridge. The second scenario is that fissure occurs in the very edge of strata near a bedrock outcrops as Fig. (5)b. Localized subsidence and its curvature is usually very low because the shallow buried bedrock slope prevented further vertical displacement. This scenario mainly distributes aside the outcropped part of the NE/NEE trending ridge between Wuxi City and Zhangji-agang City. The first two scenarios are the predominant cases in SXC area and often considered as a strong evidence of connection between differential subsidence and

earth fissure formation. In the third scenario a bedrock terrace is observed as in Fig. (5)c. The fissure locates above the slope shoulder in the upper platform. This scenario happens in the south of Changzhou City at a relatively marginal position of subsidence funnel. The last scenario is barely related to uplifting bedrock as in Fig. (5)d. Contrarily, in the case depicted in Fig. (5)d, fissure lies on the northwest side of subsidence funnel with an uplifting bedrock in the opposite side. Meanwhile, change in aquitards thickness can be observed under fissure zone. This scenario is near Zhangjiagang City.

4. Simulation result

5. Conclusion

Acknowledgements

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